Growth Systems: Expanded Mathematical Edition with Theorems

1. Governing Equation

$d^2\Phi/dt^2 + R(\Phi)d\Phi/dt = \nabla\cdot B(\Phi) + A(\Phi-Z) + G(\Phi)$

This equation partitions system evolution into inertia, damping, transport, equilibrium response, and growth. Their interaction determines global behaviour.

Theorem 1 (Oscillatory Persistence)

If R is bounded and nonzero, the system cannot converge to a static equilibrium.

Proof sketch: The second-order term ensures non-zero velocity unless fully damped, while G introduces conditional amplification.

2. Equilibrium Construction

$Z = \text{argmin} \int |\Phi - z|^2 \, dx$

Equilibrium is derived as a least-squares minimiser, ensuring it is field-dependent.

Proposition 1 (Local Optimality)

Z minimises quadratic deviation locally.

Follows from convexity of squared norm.

3. Coherence

$C = 1 / (1 + Var(v))$

Coherence measures velocity alignment.

Theorem 2 (Coherence Bound)

$0 < C \leq 1$

Follows since variance ≥ 0.

4. Growth

$G = \gamma C/(\text{entropy}+\varepsilon)(1-|\Phi-Z|/L)$

Growth depends on coherence and entropy.

Theorem 3 (Growth Threshold)

Growth occurs iff $G > R$

Defines transition between decay and amplification.

5. Energy

$E = \int(|\partial t\Phi|^2 + |\Phi\text{-}Z|^2)\, dx$

Energy combines motion and displacement.

Theorem 4 (Energy Evolution)

$dE/dt = \int(G - R|\partial t\Phi|^2)\, dx$

Energy increases only when growth exceeds dissipation.

6. Multi-Zero Dynamics

$dZ_i/dt = \Sigma \kappa(Z_j - Z_i)$

Equilibria harmonise through coupling.

Proposition 2 (Consensus)

Z_i converge to shared configuration if $\kappa > 0$

Analogous to consensus dynamics.