Statements on the Neutral Trigonometric Function

Amplitude form, balanced zeros, three-state oscillation, and directed phase approximations

Prototype: $N(x) = \sin x + \cos x = \sqrt{2}\sin(x + \pi/4)$

1. Neutral trigonometric function

Let $N(x) = \sin x + \cos x$. This function is called neutral because it combines an odd component, $\sin x$, with an even component, $\cos x$, while the total function is neither odd nor even.

$N(-x) = -\sin x + \cos x$, so in general $N(-x) \neq N(x)$ and $N(-x) \neq -N(x)$.

A trigonometric function T is called neutral if it is bounded, periodic, and neither even nor odd.

2. Amplitude form

$$N(x) = \sin x + \cos x = \sqrt{2}\sin(x + \pi/4) = \sqrt{2}\cos(x - \pi/4)$$

This shows immediately that $-\sqrt{2} \leq N(x) \leq \sqrt{2}$, so the neutral trigonometric function is a bounded oscillation of amplitude $\sqrt{2}$, centered at zero, with phase shift $\pi/4$.

In amplitude form, neutrality is expressed not as the absence of motion, but as stable oscillation around a balanced center.

3. Balanced zeros and three-state oscillation

$$\sqrt{2}\sin(x + \pi/4) = 0 \;\Rightarrow\; x = -\pi/4 + k\pi, \quad k \in \mathbb{Z}$$

These are the balanced zeros, where the odd and even components cancel exactly.

Three-state interpretation. The oscillation may be read through the three states below, with 0 as the balanced transition center.

State	Condition
Positive state	$N(x) > 0$
Neutral state	$N(x) = 0$
Negative state	$N(x) < 0$

4. Directed decimal approximations of π and phase-bounded neutral form

For decimal precision d, let π_{d-} denote the downward decimal approximation of π, and let π_{d+} denote the upward decimal approximation of π. Then $\pi_{d-} \le \pi \le \pi_{d+}$, with strict inequality whenever π is not exactly representable at that precision.

$$\pi_{d-} \le \pi \le \pi_{d+} \quad \text{and therefore} \quad \pi_{d-}/4 \le \pi/4 \le \pi_{d+}/4$$

$$\mathbf{N_{d-}(x) = \sqrt{2}\,\sin(x + \pi_{d-}/4), \quad N_{d+}(x) = \sqrt{2}\,\sin(x + \pi_{d+}/4)}$$

These give lower and upper phase approximations to the exact neutral oscillation $N(x) = \sqrt{2}\sin(x + \pi/4)$. The exact oscillation remains the central form, while the directed decimal versions act as approximative companions.

Naming suggestion. This representation may be called the phase-bounded neutral form or the directed neutral amplitude form.

The exact form lies at the neutral center between its lower and upper directed phase approximations.